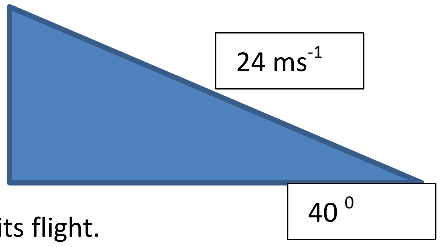
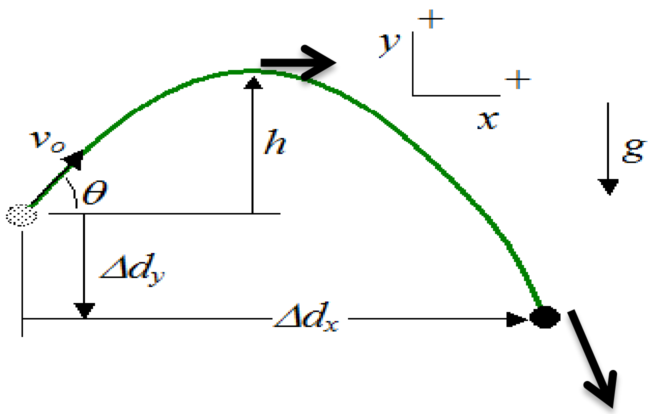
09 Motion in Fields review answers

Part A: Projectile motion:

1. A stone is thrown from a hand at a velocity of $24ms^{-1}$ and an elevation of 40^0 to the horizontal. The stone is 2m above the ground when released. Ignore air resistance.
    a. Calculate the horizontal and vertical components of velocity.
    Horizontal: 24cos(40) = $18.4ms^{-1}$
    Vertical: 24sin(40) = $15.4ms^{-1}$



    Note that for the rest of the answers we are ignoring air resistance.
    b. Calculate the time taken for the stone to reach the top of its flight.
    This is the same as time taken to reach zero vertical velocity:
    a = (v-u)/t so t = (v-u)/a = (0-15.4)/-9.81 = 1.57s
    c. Calculate the total flight time for the stone.
    There are a few ways to do this:
    Easiest is to work out the height reached then the time to descend then add this to the 1.57s taken to go up.
    Height reached = average velocity x time = (v+u)t/2 = (0+15.4)x1.57/2 = 12.1m
    Height to fall = 14.1m
    s = ut + $1/2at^2$ with u=0 because vertical velocity is zero at the high point
    s = 1/2at2 so t2 = 2s/a = 2x14.1/9.81 = 2.87 so time taken to fall is 1.70s
    Flight time = 1.57 + 1.70 =3.27s
    d. Calculate the horizontal distance travelled by the stone.
    Horizontal velocity is constant so s=vt = 18.4x3.27=60m
    e. Calculate the velocity of the stone just before landing.
    Vertical velocity = u+at = 0 + 9.81 x 1.70 = 16.7 ms^{-1}
    Horizontal velocity = 18.4 ms^{-1}. So v = 24.8 ms^{-1} at 42 degrees to horizontal.
    f. Sketch the flight path of the stone with arrows drawn to scale to represent the velocity of the stone at the start, high point and landing.



2. A bullet is fired horizontally from a gun at 500ms-1 directly at a cross on a target. If the target is 200m away how far below the cross will the bullet hit?
Time taken to reach the target = 200/500 = 0.4s
Falling distance: s = ut + $½at^2$ = 0 + $0.5x9.81x0.4^2$ = 0.78m

Part B: Gravitational field, potential and energy and orbital motion

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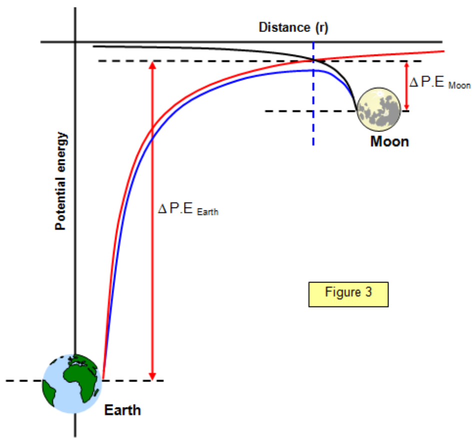
Mass of Earth = 6×10^{24}kg, Mass of Moon = 7.3×10^{22}kg

1. A satellite is in orbit 42,000 km above the centre of the Earth.
   a. Calculate the gravitational potential at this orbit height explaining why it is negative.
   $V = -GM/r = -6.67\times10^{-11} \times 6\times10^{24} / 4.2 \times 10^{7} = -9.5 \times 10^{6}$ J/kg. It is negative because it is a measurement of how much work is done to bring the object from infinity to this point.
   b. If the satellite has a mass of 150kg calculate its gravitational potential energy.
   $= 150\text{kg} \times -9.5 \times 10^{6}$ J/kg $= 1.4\times10^{9}$ J
   c. Calculate the gravitational field strength at this point and hence state the centripetal acceleration the satellite undergoes.
   $g = F/m = GM/r^2 = 6.67\times10^{-11} \times 6\times10^{24} / (4.2 \times 10^{7})^2 = 0.23$ Nkg^{-1} (0.227)
   d. Hence calculate the orbit period of this satellite in days.
   $g = a = 4\pi^2 r/T^2$ so $T^2 = 4\pi^2 r/g = 4\times \pi^2 \times 4.2 \times 10^{7}/0.227 = 7.21\times10^{9}$
   $T = 8.5\times10^{4}$s = 1 day (A geostationary satellite)
2. Derivation of the formula for escape velocity for a planet of mass M, radius r:
   a. State the energy transfer involved as an object moves completely away from a planet due only to its velocity.
   Kinetic Energy ➔ Gravitational Potential Energy
   b. State the total energy an object has when infinitely far from any other mass and when travelling at an infinitesimal velocity.
   Zero
   c. State the total energy the object must have just after launch.
   Zero
   d. Hence derive the formula for escape velocity.
   $E_p + E_k = 0$, $E_p = mV = -GMm/r$
   $-GMm/r + 1/2mv^2=0$ ➔ $v^2 = 2GM/r$ ➔ therefore $v = \sqrt{(2GM/r)}$
3. The distance from the Earth to the moon is **3.8×10^{8}m**. A point r at a distance of 3.8×10^{7}m from the moon in a direction directly towards the Earth is shown on the potential energy diagram opposite:



   a. From the graph alone state the approximate value of the gravitational field strength at this point.
   $g = -\Delta V/\Delta r$ = gradient of graph = 0
   b. Calculate the gravitational potential due to the moon and the Earth combined at this point.
   Distance to earth re = **$3.8\times10^{8}-3.8\times10^{7}$m**
   $-GM_e/r_e-GM_m/r_m$
   $-G(M_e/r_e+GM_m/r_m)$
   $-6.67\times10^{-11}(6\times10^{24}/3.42\times10^{8}+7.3\times10^{22}\text{kg}/3.8\times10^{7})$
   -1.3×10^{6} Jkg^{-1}.

c. Calculate the flight velocity required for an object, whose rockets will stop working at a distance of 7000km from the centre of the Earth to reach, this point in space.

Effect of moon is insignificant at this distance:

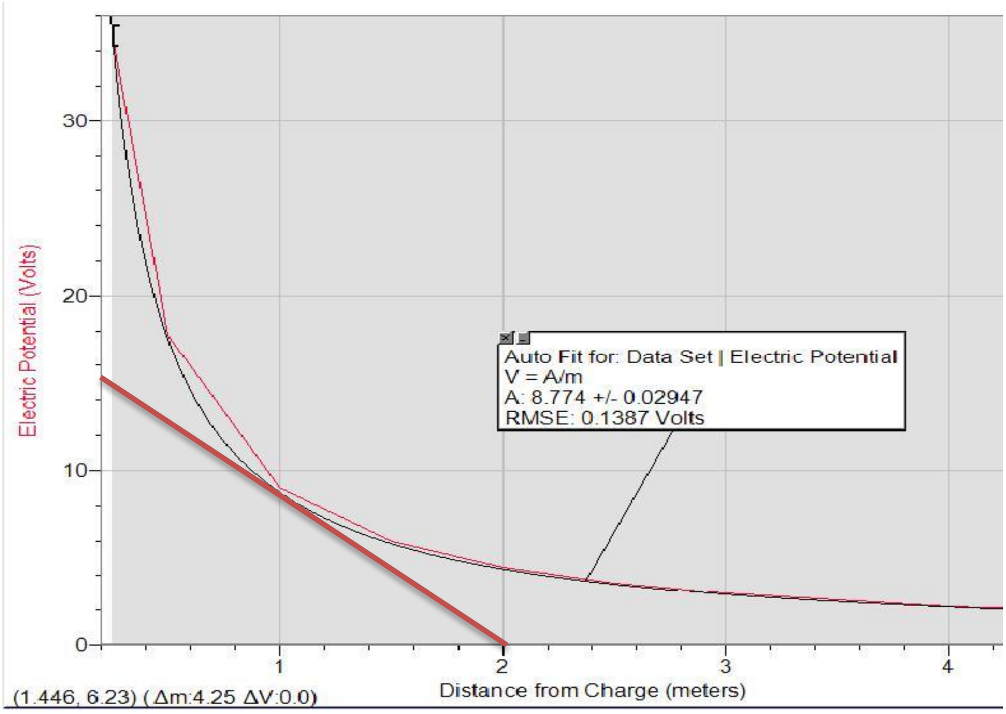
Potential at 7000km from Earth = -GM/r = $-6.67\text{x}10^{-11}\text{x}6\text{x}10^{24} / 7\text{x}10^{6} = -5.72\text{x}10^{7}$Jkg-1

½ v^2 must equal the potential difference

½ v^2 ==$-5.72\text{x}10^{7}+1.3\text{x}10^{6} = 5.59\text{x}10^{7}$ ➔ v = $1.1\text{x}10^{4}$ ms^{-1}.

Part C: Electric field, potential and energy

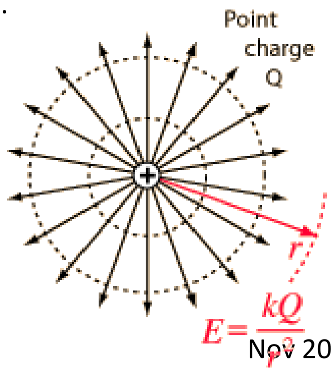
1. How much work is done to bring a charge of 1C to within $1\text{x}10^{-6}$m of a charge of 10C?
   V = kq/r = $8.99\text{x}10^{9}\text{x}10/1\text{x}10^{-6} = 9.0\text{x}10^{16}$JC-1 so $9.0\text{x}10^{16}$J of worm is done.
2. How much work is done to bring a charge of 1C to within $1\text{x}10^{-6}$m of a charge of -10C?
   $-9.0\text{x}10^{16}$J of work is done
3. The graph below shows the variation of electric potential with distance near a point charge.
   a. Use the graph to estimate the field strength at a distance of 1m.
   Field strength E = -ΔV/Δx = -Gradient of this graph = - - 15/2 = 7.5Vm^{-1}.
   b. Hence calculate the size of the charge.
   E = kq/r^2
   q = 7.5 * 1^2 / $8.99\text{x}10^{9}$ = $8.3\text{x}10^{-10}$C

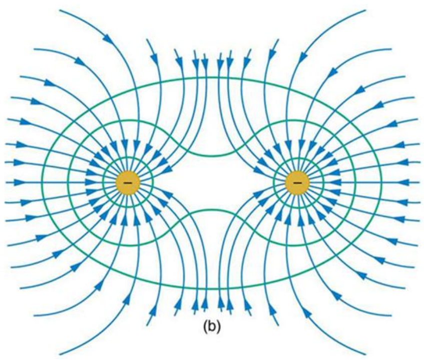


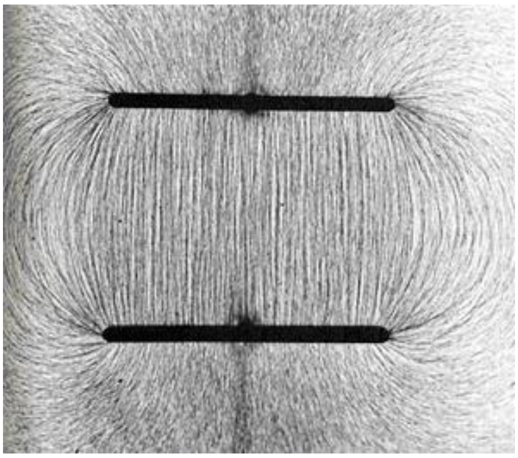
4. Sketch the equipotential surfaces and electric field lines for:

a. A point positive charge b. Two point negative charges separated by a small gap.

a.







5. The image shows electric field lines that are made visible by observing tracks of charged particles moving through a liquid. **4cm separates the plates.**
   a. If the potential difference between the plates is 6V calculate the force experienced by an ion of charge e.
      $E = \Delta V/\Delta x = 6/0.04 = 150Vm^{-1}$.
      $E = F/q$ so $F = Eq = 150e$
      $150 \times 1.6x10^{-19} = 2.4x10^{-17}N$
   b. How much work is done moving the charge all the way from one plate to another?
      i. in electron volts
         6 eV
      ii. in Joules
         6 x 1.6x10-19 = 9.6x10-19J

http://www.physics.upenn.edu/undergraduate/undergraduate-physics-labs/experiments/electric-field-and-electric-potential